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Should the Hardy-Littlewood Axioms of Collaboration be Used for Collaborative Authorship?

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ABSTRACT

It is difficult to assign authorship in bio-medical science using any fixed rule. Often strong conflicts of interest are related to two main issues: a) the rights of authorship and b) the order and position of co-authorship. The Hardy-Littlewood Rules were established on four core axioms which proposed a freedom of movement and authorship which is incompatible with most current publishing models since such co-authorship would most likely be labeled as invalid or unethical. The logic and fundament is based on an intrinsic level of trust between parties allowing complete freedom of choice. A possible ethical stumbling block may lie with the fourth axiom, which claims that all scientific papers should be published with the names of all partners, even if one or more of them had not contributed anything to the work.

Keywords: collaboration, partnerships, science writing, English skills, passion for science

COLLABORATIVE RESEARCH AND WRITING AT A GLANCE

Collaboration in science involves a network of individuals, beginning with the home lab which could include advisors, students, postdocs, research scientists, and staff. Collaboration could extend further to include the sharing of ideas, samples, equipment, and students between research groups within a department, between institutes or even transnationally or internationally. Half a century ago, it would have been common place to see a large – maybe even the majority – of papers published attributed to the efforts of a single individual. Apart from the social sciences, in the bio-medical sciences, such a feat is becoming more and more rare. This increase in collaborative-style research includes the ease of modern travel and communication, greater access to a larger data-base of information, rising emphasis on inter-disciplinary group funding, the growing complexity of many research problems being addressed today, or maybe an increase in the number of scientists who publish their research are outpacing the number of truly different ideas that can be pursued as individual efforts. Increasingly, collaboration is expanding from the realm of the laboratory and is beginning to expand into the realm of publishing and paper writing and editing (Teixeira da Silva 2011a, 2011b), which itself has a plethora of issues, both ethical and practical, but still directly pertinent to the discussion of this paper.

WHO ARE HARDY AND LITTLEWOOD?

Godfrey Harold Hardy and John Edensor Littlewood are a pair of mathematicians who worked together in Cambridge University in the first half of the twentieth century. Hardy is known by non-mathematicians for his essay from 1940 on the aesthetics of mathematics, *A Mathematician's Apology*, an epic insight into the mind of a working mathematician written for the masses. Interestingly, he was the mentor of the Indian mathematician Srinivasa Ramanujan, an impoverished Hindu genius, establishing a close collaboration in what Hardy was to term "the one romantic incident in my life." (Kanigel 1991; Freudenberger 2007; Web-site 1) Oddly, there is no description as to why Hardy-Littlewood Rules were never termed the Hardy-Ramanujan Rules or whether the Hardy-Littlewood Rules were ever applied to the Hardy-Ramanujan relationship. Interestingly, a series of papers *Partitio numerorum* used the Hardy-Littlewood-Ramanujan analytical method and it would be curious to know the actual relative contribution of these three mathematicians to this and to other theorems. However, Hardy's collaboration with Littlewood is among the most successful and famous collaborations in mathematical history. So much so that in a 1947 lecture, the Danish mathematician Harald Bohr reported a colleague as saying, "Nowadays, there are only three really great English mathematicians: Hardy, Littlewood, and Hardy–Littlewood." (Bohr 1952) Hardy is also known for formulating the Hardy–Weinberg principle, a basic and important principle of population genetics.

Littlewood, the lesser known of the two, coined Littlewood's law, which states that individuals can expect miracles to happen to them, at the rate of about one per month (Bollobás 1986).

This successful partnership resulted in the creation of the Hardy–Littlewood circle method, which is one of the most frequently used techniques of analytic number theory, although the idea is generally attributed to Hardy (Vaughan 1997). The Hardy–Littlewood tauberian theorem is a tauberian theorem related to the asymptotics of the partial sums of a series with the asymptotics of its Abel summation (Hazewinkel 2001). The Hardy–Littlewood Maximal Inequality can prove the Lebesgue differentiation theorem, the Rademacher differentiation theorem and Fatou's theorem on nontangential convergence (Hardy and Littlewood 1932; Melas 2003).

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WHY ARE THE HARDY-LITTLEWOOD AXIOMS IMPORTANT?

What was unique about Hardy and Littlewood was that they had established a long and harmonious mathematical collaboration based on the following rules or axioms, spelt our by Bohr (1952), and paraphrased here as: Rule 1: what they wrote to each other was completely indifferent whether what they said was right or wrong. As Hardy put it, otherwise they could not write completely as they pleased since they would have to feel a certain responsibility thereby; Rule 2: They were under no obligation to reply, or even to read, let alone answer, any letter (or communication) that one may have sent to the other. As they said, it might be that the recipient of the letter would prefer not to work at that particular time, or perhaps that he was just then interested in other problems; Rule 3: They had to try not try to think about the same details, and in fact, it was preferable that they not do so; Rule 4 (the most critical and thought-provoking): To avoid any arguments, all scientific papers would be published with both names, even if one of them had not contributed anything to the work.

In mathematics, the Hardy-Littlewood rule is used. That is, authors are alphabetically ordered and everyone gets an equal share of credit independent to their actual contribution. Graham Fan Chun provides some advice on what not to do "If you have any bad feeling about sharing the work or the credit, don't collaborate. In mathematics, it is quite okay to do your research independently. (Unlike other areas, you are not obliged to include the person who fund[s] your research.) If the collaboration already has started, the Hardy-Littlewood rule says that it stays a joint work even if the contribution is not of the same proportion. You have a choice of not to collaborate the next time. (If you have many ideas, one paper doesn't matter. If you don't have many ideas, then it really doesn't matter.) You might miss the opportunity for collaboration which can enhance your research and enrich your life. Such opportunity is actually not so easy to cultivate but worths all the efforts involved." (Web-site 2) The question arises: if a mathematician were to want to opt out of the option of using the Hardy-Littlewood rules, is that possible if submitting to a mathematics journal? Under normal circumstances choices of collaboration are optional and usual mutual consensus results in the cancellation of such a collaboration.

Cartwright (1981) conjectures that these rules were actually agreed on by Littlewood and Hardy in 1912. How could such a prolific mathematician as Littlewood have his collected papers published in only two volumes? This is because the large Hardy-Littlewood collection of papers appears in Hardy's collected works of five volumes published by Cambridge University Press. J.C. Burkill describes their partnership as follows: "Normally Littlewood would make the penultimate version of a paper, with a skeleton of all the essential mathematics, simplifying and abbreviating in notation clear to Hardy. Hardy would add what they called the 'gas' and write the paper in the elegant prose of which he was a master. Littlewood's own style, in its clarity and brevity, was equally magisterial." (Burkill and Burkill 1970).

The Hardy-Littlewood collaboration was interrupted by World War I.

DEBATING THE HARDY-LITTLEWOOD RULES FOR COLLABORATIVE PUBLISHING

To the person who reads this paper for the first time, undoubtedly they would remark "Hardy and Littlewood had incredible trust in one another." These two scientists created this basic set of rules to cement the trust, making their collaboration productive and peaceful. Their first rule allowed them to respect each other by being accepting of each other's criticisms without stopping them from focusing on the work. Basically, no matter what they wrote would neither be considered right nor wrong. The second and third rule gave each member the peace of mind to do their own independent work without worrying about what the other was doing, encouraging however, each to focus on different aspects to avoid an overlap and conflict. Neither was there the pressure of expected deadlines or forced labor, nor was there a neck-tightening noose binding them to schedules or objective frame-works. The final rule would bring the fruit to their peaceful accord by guaranteeing that no matter what was produced, or what was created, it would guarantee authorship, fame and an expanded legacy, even without having actually participated in actual research.

How could such a harmonious professional writing relationship be established on such incredibly naive and potentially negatively interpreted axioms?

These rules were created, in essence, to guarantee their personal freedoms within the realm of a professional working relationship. At first sight to a layman, the rules may appear lax or too free, particularly to a scientist who is always surrounded by limits and rules, but closer scrutiny reveals a responsible freedom based on trust, the basis for a working relationship. Regarding Rule 1, if a scientist were never to make mistakes, he/she would not work creatively. Conversely, by working creatively, at least sometimes a scientist makes mistakes and it would be important to know that a scientist can work without the fear of being judged by anyone for making a mistake. Such mistakes are an important key for persisting. Rule 2 poses an interesting question because it gives the freedom for both (or all) parties but it assumes that none of the collaborators will feel nervous if an answer is lacking. Sometimes, however, it is enough if a question for someone can be posed and the answer becomes naturally clear to him/herself – without any answer. In theory, Rule 4 would pose the greatest danger to the scientific community because it would challenge the validity of authorship. However, in practice, it is most likely that a collaboration would have initially been established on some core principles like common interest, passion, and trust, validating Rules 1-3. Then, Rule 4 would theoretically never apply because a collaboration where one does something and the other does nothing would be a marriage of fools, without any advantage to both. So, a common desire to strive for common goals, within a framework of trust, would ensure that Rule 4 would never be abused, even if the freedom to do so existed.

HARDY-LITTLEWOOD RULES IN THE MATHEMATICAL SCIENTISTS

The Hardy-Littlewood Rules appear to have a dichotomy at present: the perceived use according to a silently understood convention, and the current actual application to the mathematical sciences in pure and applied mathematics journals. From a random sample of mathematics journals listed at Web-site 3, we discovered that at least 10 did not follow the rules, or where there were exceptions to the rules within the journals, irrespective of the publisher or the Impact factor of the journal, leading us to believe that some possibilities are at play: a) convention is not universally respected; b) convention is fading as is perhaps the trust in the Hardy-Littlewood Rules; c) convention might simply be understood among US and EU mathematicians; d) Who has the right to chose and decide the order of authors for the mathematical sciences: journal policies or individual groups (of scientists)? Independent of the actual reason for the erosion of the Hardy-Littlewood Rules in pure and applied mathematics journals, it does reflect a weakening trust among mathematics scientists which will lead, eventually, to the authorship and ethical issues associated with it that are currently plaguing the bio-medical field (Teixeira da Silva 2011c). The ethical issues and conflicts of interest would become more understandable if we were to imagine a hypothetical case where there are three co-authors (randomly Adams, Lilly and Zeewolf) and if Lilly had done most of the work or Adams had done none.

PRO'S AND CON'S OF THE HARDY-LITTLEWOOD RULES

The positive and negative sides of the Hardy-Littlewood collaboration rules are undoubtedly attractive to some and repulsive to others. Below, we try to outline as balanced a view as possible.

Pro's:

1. A simplified, stress-free and trust-based relationship. But there is a con.
2. There are no risks, only gains. But there is a con.
3. When working with others in research, people implicitly tend to take on different natural paired contrasting roles, often balancing strengths and weaknesses. When taking on these different roles they can greatly enhance the process, and its dynamic. Consequently, it may be worthwhile for people to explicitly take on these roles (and switch off from time to time). It avoids conflicts and stress and provides the freedom of space to work and the freedom of mind to think. The contrasting roles that could be assumed are: optimist/pessimist, writer/editor, or implementer/debugger.
4. In the long term, a scientist will be known for their body of work over their lifetime, and people down the road will see what they have done over a number of projects and over the years. This is termed a scientist's legacy. If a scientist works alone they might only be able to achieve a fraction of what could have been achieved had they collaborated (broadly-speaking). Thus their legacy potentially becomes maximized through collaboration, even more so when based on the Hardy-Littlewood Rules which gives exponential credit for minimal work or for well-devised partnerships.
5. The alphabetical ordering of authors is inherent to the systems community, including the mathematical sciences. In contrast, a non-alphabetical order – as is the case with most bio-medical journals – is corrosive: whenever there is a (sub)cast of authors who have equal credit due, it forces hairs to be split as to how these authors should be listed, and in what order. This does not add anything to the group dynamic nor does it reflect any actual difference in contribution. Non-alphabetic order is sometimes required when it is obligatory to put adviser or head of the lab as co-authors while their only contribution is proofreading of the paper before submission. Therefore, in a journal that publishes papers in a non-alphabetical order, there is a strong possibility of conflict and erosion of the group dynamic, clearly avoided when using the Hardy-Littlewood Rules. For example, if a scientist is judged by their publication efforts based on how many papers are written with their name as first author, this would result in intensive competition, and would potentially lead to rivalry among authors who would vie for different places in the rank of authors. Such a negative atmosphere may hinder the positive development of science and may result in the ranking of a researcher mainly on the basis of their position in a paper or in a research team but not on the basis of their scientific performance or ability. The Hardy-Littlewood rules would avoid such negative conflicts and could re-emphasize the importance of creative work by providing the freedom of each individual to shine while de-emphasizing the conflicts that may be caused by competition between parties. The Hardy-Littlewood Rules (in the case of non-alphabetical ordering) in some cases tries to conserve a traditional system of authorship which was suitable in the past in which a large amount of research work resulted mainly from the efforts of individuals and in which the measurement of an individual's performance was emphasized. However, research in modern labs is the product of the performance of a group and very often the exact individual contribution cannot be measured or quantified.

Con's:

1. If your family name/surname starts with a letter of the alphabet somewhere near the end such as Xu or Zhang, your name is likely to always appears last (the upside is that many readers might consider you to be the leader or principal supervisor). Those individuals whose family name heads the alphabet such as Adams, could invariably always be referenced as Adams *et al*. in all papers (> 2 authors), thus indirectly (automatically) eulogizing Adams.
2. One or more collaborators, especially in large teams, might not pull their weight, leading to an imbalance of efforts, and potential laziness and entry into the "comfort zone" by some. Who then gets the credit? Is it important? Finally a paper that is published in Nature is Nature, after all, but there is the potential for a *blazé* attitude by some members, who may be reaping recognition unfairly. How would one limit the size of the group without appearing to be discriminatory even if it were possible to show that a member had in fact not done anything?
3. The premise is that the relationship is based on trust, which is probably fortified over time. In a competitive environment where several collaborators may be competing for the same equipment, the same working space, the same funding or the same job, rivalry has the potential of inducing a very bitter after-taste and rapidly eroding away that trust. In this case, surely it is better to maintain a modest level of rivalry to ensure the quality of work, despite the slightly foul smell in the air.
4. In the mathematical sciences where authorship is universally established as being alphabetical (although this appears to be rapidly changing), there are no risks, only gains. This could prove more problematic in the bio-medical sciences where such a rule is not specified, or in fringe journals that have overlapping fields of study and where the editorial board or publisher is wishy-washy about its policies governing authorship. In particular with regard to credit. It is difficult – if not eventually impossible – to foster an environment where one can collaborate with others easily and naturally when one or more members are always worried about who will get the credit.
5. A simplified, stress-free and trust-based relationship. However, can data and experimental execution be trusted? How is trust assessed and verified?
6. Most bio-medical journals follow different rules of engagement and ethics regarding authorship, although following the rules of the International Committee of Medical Journal Editors (ICMJE 2006) (broadly) clash with the definitions provided by other ethical bodies or even publishers such as Elsevier (discussed in detail in Teixeira da Silva 2011a, 2001b, 2011d), a difference existence in a simple difference in prepositions: or vs and. Most bio-medical journals might not be receptive to such a lax system that is unable to identify the active participation of each co-author, and hence their responsibility as scientists towards society (Teixeira da Silva 2011c). Creative and free collaboration would thus be stymied by artificial – to some extent unrealistic – ethical parameters set by publishers or journals.
7. This is an almost socialist system of authorship with equal distribution of laurels. There is no recognition of individual strengths or merits and all are herded together to emphasize the strength of the collective, the group, rather than the individual. To avoid this, members of a group could decide when or when not to apply the Hardy-Littlewood collaboration rules, i.e. to only apply the rule when indeed all members actively participated. However, selective use of the Hardy-Littlewood rules would fray the fundamental basis of what Hardy and Littlewood wanted to achieve, which was to ensure co-authorship for all papers under the basis of a trustworthy relationship. The weakness of the Hardy-Little-

wood rules is that trust becomes diluted as the group size increases. And where trust breaks down between any two members of a group, the effectiveness of the Hardy-Littlewood collaboration rules is diluted.

CONCLUSIONS AND PERSPECTIVES

Publishing is the climax of a scientist's career where, in print or online, that author's legacy is marked forever, like an imprint in the sand or the snow. In this day and age of highly competitive publishing, collaboration is key to advancing the scientific agenda and ensuring a stake in the wider sea of scientific findings, thus ensuring social respect and power (Teixeira da Silva 2011c). Authorship is – and most likely will always be – a hot-bed of conflict and discourse because it is founded on the basis of the ability – or not – to manage human relations effectively and to master the concept of the dynamic team. Resolving the issue of authorship would require a set of rules or guidelines that would assist the decision-making of the scientist regarding authorship – who, in what position and with what weighting – without imposing any further undue stress on an already battle-prone career. However, such a set of rules should provide the scientist with the freedom to act while still ensuring a heightened legacy. In the mathematical sciences the Hardy-Littlewood Rules begin to provide such a platform, but these rules are far from imaginable in the biomedical sciences, which is so ferociously fighting for funding and market space at any cost: 1000 dogs to a single bone, so to speak. Unless there is a high level of trust between collaborators such that a peaceful agreement would satisfy all parties *ad infinitum*, such an arrangement could be terminated when it no longer becomes mutually beneficial. Naturally, there are the ethical parameters associated with the Hardy-Littlewood Rules since they would clash, head-on, with most of the strongly imposed ethical guidelines as determined by a handful of ethical bodies and publishers. At base, productive collaborations that do occur in science are driven by the shared curiosity and excitement of the collaborators, and not by any obvious need for individuals to contribute their pieces to a puzzle. Possibly, this is the real meaning behind the Hardy-Littlewood axioms for a successful collaboration. Personally, I (JTdS) could go either way, depending on the partner I am dealing with, i.e. choice of the use of the Hardy-Littlewood Rules is not a viable choice for all scientists and should be used very selectively, i.e. it should be a voluntary process determined by the collaborating partners and not by the publishers.

QUOTES BY HARDY AND LITTLEWOOD

By GH Hardy

"It is never worth a first class man's time to express a majority opinion. By definition, there are plenty of others to do that." "A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas." "Nothing I have ever done is of the slightest practical use." "If I could prove by logic that you would die in five minutes, I should be sorry you were going to die, but my sorrow would be very much mitigated by pleasure in the proof."

By JE Littlewood

"I took things as they came; the game we were playing came easily to me, and I even felt a sort of satisfaction in successful craftsmanship." "In passing, I firmly believe that research should be offset by a certain amount of teaching, if only as change from the agony of research. The trouble, however, I freely admit, is that in practice you get either no teaching, or else far too much." "A good mathematical joke is better, and better mathematics, than a dozen mediocre papers." "The surprising thing about this paper is that a man who could write it would." "Try a hard problem. You may not solve it, but you will prove something else."

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